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FRACTIONS

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Purpose

This SMART PACK is designed to help you do various calculations involving fractions and mixed numbers.

Objectives:

Upon completion of this SMART PACK, you should be able to:

1. Add fractions and mixed numbers
2. Subtract fractions and mixed numbers
3. Mulitply fractions and mixed numbers
4. Divide fractions and mixed numbers

Overview

A fraction is a portion of a whole amount. Fractions are used routinely in scientific as well as everyday endeavours. A good understanding of how to use fractions is necessary in order to succeed in scientific and technical work.

PART I - Fractions - Proper, Improper and Mixed

What is a fraction? A fraction is a part of something. For example, 3 cents is $\frac{3}{10}$ of a dime. The number on top of the fraction bar is called the numerator; the number below the fraction bar is called the denominator. If the numerator is smaller than the denominator, the fraction is a proper fraction; if the numerator is equal to or larger than the denominator, the fraction is an improper fraction. For example, $\frac{3}{10}$ is proper, but $\frac{10}{3}$ is improper.

A fraction can be reduced if both the numerator and denominator can be divided by the same number. For example, in the fraction $\frac{12}{15}$, both the 12 and the 15 can be divided by 3. When this is done, $\frac{12}{15}$ is said to be reduced to $\frac{4}{5}$. $\frac{12}{15}$ is equal to $\frac{4}{5}$, or $\frac{12}{15}=\frac{4}{5}$.

In the reverse procedure from that above, a fraction can be raised to higher terms if both the numerator and denominator are multiplied by the same number. For example, if the numerator and the denominator of the fraction $\frac{2}{3}$ are each multiplied by 4, the resulting fraction is $\frac{8}{12}$. $\frac{2}{3}=\frac{8}{12}$

An improper fraction can be changed to a mixed number by dividing the denominator into the numerator. If 2 is divided into 15 in the improper fraction $\frac{15}{2}$, the result is 7 with a remainder of 1. This can be written $7\frac{1}{2}$. $7\frac{1}{2}$ is a mixed number.

In the reverse procedure, a mixed number can be changed to an improper fraction by multiplying the denominator of the fraction part by the whole number part, then adding the numerator of the fraction part. Finally place that total over the denominator. For example, to change $8\frac{2}{3}$ to an improper fraction, multiply 3 by 8 and then add 2. Place the result, 26, over 3. $8\frac{2}{3}=\frac{26}{3}$.

Practice Problems - Set I

1. Reduce $\frac{6}{8}$
2. Reduce $\frac{10}{25}$
3. Change $\frac{5}{8}=\frac{?}{24}$
4. Change $\frac{2}{5}=\frac{?}{20}$
5. Change $\frac{11}{5}$ to a mixed number
6. Change to $\frac{23}{3}$ a mixed number
7. Change to $6\frac{1}{4}$ an improper fraction
8. Change $10\frac{2}{7}$ to an improper fraction

PART II - Addition and subtraction of fractions and mixed numbers

To add or subtract fractions, the denominators must be the same. For example, $\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$. The numerators are added and placed over the denominator. Similarly, $\frac{9}{11}-\frac{4}{11}=\frac{5}{11}$.

If the sum (the answer you get when you add) or the difference (the answer you get when you subtract) can be reduced or written as a mixed number, then you should do that. For example, $\frac{7}{8}+\frac{5}{8}=\frac{12}{8}=1\frac{4}{8}=1\frac{1}{2}$. Or $\frac{9}{10}-\frac{1}{10}=\frac{8}{10}=\frac{4}{5}$.

What happens if the fractions you want to add or subtract don't have the same denominator? Not to worry! It is always possible to find a common denominator (preferably the lowest common denominator, or LCD) and then proceed as usual.

When adding or subtracting fractions with different denominators decide if the largest denominator is a multiple of all of the other denominators. For example, in the problem $\frac{2}{3}+\frac{1}{2}+\frac{5}{6}$, 6 is a multiple of 3 and 2 (another way of saying that 3 and 2 are divisors of 6). In this case, 6 is the LCD. The next step would be to change $\frac{2}{3}$ and $\frac{1}{2}$ to sixths: $\frac{2}{3}=\frac{4}{6}$ and $\frac{1}{2}=\frac{3}{6}$, so our problem now becomes $\frac{4}{6}+\frac{3}{6}+\frac{5}{6}=\frac{12}{6}=2$.

Suppose you have this problem: $\frac{7}{8}-\frac{1}{6}$. Clearly 8 is not a multiple of 6 (6 is not a divisor of 8), so you need another method of finding a common denominator or an LCD. One way of finding a common denominator would be to simply multiply 8 by 6. 48 will certainly work! Since $\frac{7}{8}=\frac{42}{48}$ and $\frac{1}{6}=\frac{8}{48}$, we have $\frac{42}{48}-\frac{8}{48}=\frac{34}{48}=\frac{17}{24}$. Another way to find a common denominator (this method will get you the LCD) is to list the multiples of 8 and 6 until you get a match:

multiples of 8: 8, 16, 24, 32, etc.

multiples of 6: 6, 12, 18, 24, etc.

See that 24 is the first match obtained and is therefore the LCD! Now change $\frac{7}{8}$ to $\frac{21}{24}$ and $\frac{1}{6}$ to $\frac{4}{24}$. We now have $\frac{21}{24}-\frac{4}{24}=\frac{17}{24}$.

More examples:

$$3\frac{3}{4}+2\frac{2}{5}=3\frac{15}{20}+2\frac{8}{20}=5\frac{23}{20}=5+1\frac{3}{20}=6\frac{3}{20}$$

$$5\frac{5}{6}-2\frac{1}{3}=5\frac{5}{6}-2\frac{2}{6}=3\frac{3}{6}=3\frac{1}{2}$$

Sometimes when subtracting mixed numbers, we have to re-express from the whole number part in order to subtract the fractions. For example: $9\frac{5}{12}-3\frac{7}{8}$. Do you see that you can't subtract $\frac{7}{8}$ from $\frac{5}{12}$? If 1 from the 9 is rewritten as $\frac{12}{12}$ then is added to the $\frac{5}{12}$ we now have $8\frac{17}{12}$. The next step is to find a common denominator and change the fractions. The LCD is 24. $8\frac{17}{12}=8\frac{34}{24}$ and $3\frac{7}{8}=3\frac{21}{24}$. So now we can do the subtraction: $8\frac{34}{24}-3\frac{21}{24}=5\frac{13}{24}$.

Practice Problems - Set II

Perform the indicated operation.

1. $\frac{13}{16}-\frac{3}{16}$
2. $\frac{1}{4}+\frac{5}{6}+\frac{5}{12}$
3. $3\frac{9}{10}-1\frac{1}{5}$
4. $9\frac{5}{12}+1\frac{4}{9}$
5. $8\frac{1}{2}+3\frac{3}{4}+2\frac{2}{3}$
6. $12\frac{1}{5}-8\frac{4}{5}$
7. $15-3\frac{1}{8}$
8. $27\frac{1}{9}-20\frac{5}{6}$

PART III - Multiplication and division of fractions and mixed numbers

Basically, the rule for multiplication is: multiply the numerators, multiply the denominators! For example, $\frac{2}{3}\cdot\frac{4}{5}=\frac{8}{15}$. However, if you can cancel before multiplying, you can make your work a bit simpler. Canceling is like reducing. For example, look at $\frac{4}{3}\cdot\frac{15}{16}$. You could multiply $4\cdot 15$ to get 60 in the numerator, and multiply $3\cdot 16$ to get 48 in the denominator. The answer would then be $\frac{60}{48}$, which now you have to reduce, then write as a mixed number. If you canceled first:

$$\frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}}\cdot\frac{\overset{5}{\cancel{15}}}{\underset{4}{\cancel{16}}}=\frac{5}{4}=1\frac{1}{4}$$

See? No reducing was necessary! Here's another example:

$$\frac{\overset{1}{\cancel{15}}}{\underset{\underset{1}{4}}{\cancel{28}}}\cdot\frac{\overset{1}{\cancel{7}}}{16}\cdot\frac{\overset{\overset{1}{\cancel{3}}}{\cancel{12}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{45}}}=\frac{1}{16}$$

It makes no difference what order you do the canceling or reducing:

$$\frac{\overset{1}{\cancel{15}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{28}}} \cdot \frac{\overset{1}{\cancel{7}}}{16} \cdot \frac{\overset{\overset{1}{\cancel{3}}}{\cancel{12}}}{\underset{\underset{1}{\cancel{15}}}{\cancel{45}}} = \frac{1}{16}$$

If you are multiplying two or more mixed numbers or fractions and mixed numbers, the mixed numbers must be changed to improper fractions. For example:

$$\frac{2}{7} \cdot 2\frac{5}{8} = \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{7}}} \cdot \frac{\overset{3}{\cancel{21}}}{\underset{4}{\cancel{8}}} = \frac{3}{4}$$

More examples:

$$\frac{7}{8} \cdot 24 = \frac{7}{\underset{1}{\cancel{8}}} \cdot \frac{\overset{3}{\cancel{24}}}{1} = \frac{21}{1} = 21$$

$$3\frac{3}{4} \cdot \frac{8}{9} \cdot 6 = \frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{4}}} \cdot \frac{\overset{2}{\cancel{8}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{9}}} \cdot \frac{\overset{2}{\cancel{6}}}{1} = \frac{20}{1} = 20$$

$$2\frac{2}{5} \cdot 3\frac{3}{8} \cdot 2\frac{7}{9} = \frac{\overset{3}{\cancel{12}}}{\underset{1}{\cancel{5}}} \cdot \frac{\overset{3}{\cancel{27}}}{\underset{2}{\cancel{8}}} \cdot \frac{\overset{5}{\cancel{25}}}{\underset{1}{\cancel{9}}} = \frac{45}{2} = 22\frac{1}{2}$$

To divide one fraction by another, invert the divisor (the number to the right of the ÷ sign), then multiply as usual. For example:

$$\frac{4}{9} \div \frac{2}{3} = \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{9}}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} = \frac{2}{3}$$

As with multiplication, if you are dividing mixed numbers, they must be changed to improper fractions first:

$$\frac{14}{15} \div 6 = \frac{12}{5} \div \frac{6}{1} = \frac{\overset{2}{\cancel{12}}}{5} \cdot \frac{1}{\underset{1}{\cancel{6}}} = \frac{2}{5}$$

$$\frac{14}{15} \div 1\frac{1}{6} = \frac{14}{15} \div \frac{7}{6} = \frac{\overset{2}{\cancel{14}}}{\underset{5}{\cancel{15}}} \cdot \frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{7}}} = \frac{4}{5}$$

$$12 \div 1\frac{3}{5} = \frac{12}{1} \div \frac{8}{5} = \frac{\overset{3}{\cancel{12}}}{1} \cdot \frac{5}{\underset{2}{\cancel{8}}} = \frac{15}{2} = 7\frac{1}{2}$$

$$3\frac{3}{7} \div 1\frac{11}{21} = \frac{24}{7} \div \frac{32}{21} = \frac{\overset{3}{\cancel{24}}}{\underset{1}{\cancel{7}}} \cdot \frac{\overset{3}{\cancel{21}}}{\underset{4}{\cancel{32}}} = \frac{9}{4} = 2\frac{1}{4}$$

Practice Problems - Set III

Perform the indicated operation. Make sure your answer is reduced or changed to a mixed number.

1. $\frac{7}{8} \cdot \frac{9}{14} \cdot \frac{5}{6}$

2. $32 \cdot \frac{7}{16}$

3. $4\frac{1}{2} \cdot \frac{5}{6}$

4. $2\frac{1}{3} \cdot 1\frac{1}{5}$

5. $\frac{16}{21} \div \frac{3}{4}$

6. $\frac{24}{25} \div 40$

7. $9 \div 1\frac{7}{8}$

8. $10\frac{2}{3} \div 2\frac{2}{3}$

Answer Key

Practice Problems - Set I

1. $\frac{3}{4}$
2. $\frac{2}{5}$
3. 15
4. 8
5. $\frac{11}{5} = 2\frac{1}{5}$
6. $\frac{23}{3} = 7\frac{2}{3}$
7. $6\frac{1}{4} = \frac{25}{4}$
8. $10\frac{2}{7} = \frac{72}{7}$

Practice Problems - Set II

1. $\frac{10}{16} = \frac{5}{8}$
2. $\frac{18}{12} = 1\frac{6}{12} = 1\frac{1}{2}$
3. $2\frac{7}{10}$
4. $10\frac{31}{36}$
5. $13\frac{23}{12} = 14\frac{11}{12}$
6. $3\frac{2}{5}$
7. $11\frac{7}{8}$
8. $6\frac{5}{18}$

Practice Problems - Set III

1. $\frac{15}{32}$
2. 14
3. $3\frac{3}{4}$
4. $2\frac{4}{5}$
5. $1\frac{1}{63}$
6. $\frac{3}{125}$
7. $4\frac{4}{5}$
8. 4